Design Considerations for a Direct RF Sampling Mixer
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Abstract—This brief presents a detailed time-domain and frequency-domain analysis of a direct RF sampling mixer. Design considerations such as incomplete charge sharing and large signal nonlinearity are addressed. An accurate frequency-domain transfer function is derived. Estimation of noise figure is given. The analysis applies to the design of sub-sampling mixers that have become important for software-defined radio and analog-to-digital converter.

Index Terms—Charge sharing, embedded filtering, noise, nonlinearity, sampling mixer, windowed integration sampler.

I. INTRODUCTION

THE PERVERSIVE wireless applications of many communication standards with different carrier frequencies, channel bandwidth, and modulation schemes have motivated the industry to look for multiband multistandard solutions to enable anywhere anytime connectivity [1]. The software-defined radio (SDR) that Mitola proposed is the ultimate solution [2]. However, to cover all standards up to 5 GHz, a 12-bit and 10-GHz sample rate analog-to-digital converter (ADC) is needed for an ideal software-defined radio receiver (SDRX). This approach will remain impractical in the near future [1]. Recently, important technology breakthroughs in SDRX, such as the software-defined radio multiband receiver of University of California at Los Angeles (UCLA) [1], and the fully integrated quad-band GSM/GPRS receiver of Texas Instruments (TI) [3], solve the problem in a similar way. The TI solution uses direct RF sampling mixer to downconvert the wanted RF bands to around dc. In addition, it simultaneously provides anti-aliasing filtering and sampling functionalities, and by reducing the interferers and blockers it relaxes the stringent dynamic range requirements for the ADC. Moreover, the direct RF sampling mixer pushes analog-to-digital conversion close to the antenna, enabling low-cost and low-power design that exploits sophisticated digital-signal-processing (DSP) algorithms. This avoids analog signal processing influenced by small voltage headroom of the deep-sub-micrometer CMOS technologies [4]. The working principle of the direct RF sampling mixer was discussed in [3], [4], and [5]. This brief presents design considerations that have not been previously reported in literature. These are the effect of incomplete charge sharing; large signal nonlinearity, and the lack of a detailed system transfer function. Furthermore, modifications of noise figure estimation are introduced, because noise sources are strongly shaped by low-pass filters in the direct RF sampling mixer. This is due to the high sampling frequency and large decimation factor.

II. BRIEF REVIEW OF WINDOWED INTEGRATION SAMPLER

The core of a direct RF sampling mixer is a charge sampling mixer [6] or windowed integration sampler (WIS) [1]. The transconductor-capacitor (\(G_m-C\)) structure of the WIS greatly reduces the influence of the nonlinear drain–source resistance of MOS switches. As it is very hard to switch off a current source, the structure shown in Fig. 1 is used, which also gives the transconductor a constant load at any time [7]. During tracking, the output voltage can be described by

\[
C \frac{dV_o(t)}{dt} = G_m V_i(t).
\]

(1)

At any sampling instant, the following relationship exists:

\[
x(t_n) = V_o(t_n) - V_o(t_n - T_{on}) = \frac{G_m}{C} \int_{t_n=0}^{t_n} V_i(t) dt
\]

(2)

where \(x(t_n)\) is a virtual signal defined as change of the sampled output voltage against its previous sample. \(x(t_n)\) is actually the sampled voltage on the sampling capacitor if the capacitor is reset in every sampling period. Let

\[
x(t) = \frac{G_m}{C} \int (V_i(t) - V_i(t - T_{on})) dt.
\]

(3)

At any sampling instant \(t_n\), (3) is exactly the same as (2), although at any other time they may differ considerably. Therefore, a linear transfer function from the continuous-time to the sampled discrete-time is yielded as

\[
SH(f) = \frac{G_m}{C} \frac{1-e^{-j2\pi f T_{on}}}{j2\pi f} = e^{-j\pi f T_{on}} \frac{G_m T_{on}}{C} \sin(\pi f T_{on})
\]

(4)
If the input signal is a bandpass signal, more specifically a QAM signal whose spectrum is
\[ \frac{1}{2} [R(f + f_0) + R(f - f_0)] + \frac{j}{2} [M(f - f_0) - M(f + f_0)] \]  
(5)
where \( R(f) \) and \( M(f) \) are baseband signals with a bandwidth of \( f_b \). According to the sampling theorem, the discrete Fourier transform (DFT) of the sampled output signal is
\[ \frac{1}{T_s} \sum_{k=-\infty}^{\infty} e^{-j2\pi knT_s/T_s} V_i(f - kf_s) SH(f - kf_s). \]  
(6)
This equation shows, through sampling the spectra at integer multiples of the sampling frequency are downconverted to dc. For an approximation, we assume the input signal is narrow band
\[ |f| \leq \frac{f_0}{2} \ll f_s = \frac{f_0}{N} \]  
(7)
where \( N \) is a positive integer, \( f_0 \) is the carrier frequency, and \( f_s \) is the sampling frequency. When the integration time \( T_{on} \) is one half of the sampling period, approximation for \( SH(f) \) becomes
\[ SH(f \pm f_0) \approx \begin{cases} 0, & \text{when } N \text{ is even} \\ \pm \frac{G_m}{\pi f_0}, & \text{when } N \text{ is odd}. \end{cases} \]  
(8)
In this case, the ratio of the sampling frequency must be an odd integer division of the carrier frequency, because the windowed integration introduces zeros at the carrier frequency and its integer multiples if \( N \) is even. The integration window \( T_{on} \) and the sampling phase \( T_{ph} \) can be manipulated to provide embedded anti-aliasing filtering to reduce wideband noise, interferers and blockers before they alias into the baseband [1], [8]. If the capacitor is reset in every period, the spectrum of a sampled quadrature amplitude modulated (QAM) signal in the first Nyquist band is obtained as
\[ V_o(f) = \frac{G_m}{\pi f_0 C} \begin{cases} -M(f), & \text{when } T_{ph} = 0 \\ M(f), & \text{when } T_{ph} = 0.5T_s \\ (-1)^N R(f), & \text{when } T_{ph} = 0.25T_s \\ (-1)^{N+1} R(f), & \text{when } T_{ph} = 0.75T_s. \end{cases} \]  
(9)
With proper sampling phases, the WIS circuits not only demodulate a QAM signal into the first Nyquist band, but also separate its quadrature contents into pseudo-differential signals. The sampled signal has a symbol rate of the sampling frequency, thus a lower sampling rate is desirable for the back-end digital baseband processor. However, noise folding-back requires a high sampling frequency.

III. DIRECT RF SAMPLING MIXER

The noise sources in a sampling mixer include thermal noise, switching noise and 1/f noise [9]. When a sampling clock with low phase noise and sharp edges is used, the dominant noise added by the sampling mixer is thermal noise and the folding-back of wide-band noise from the input signal. A direct RF sampling mixer resets the rotating capacitors periodically and the noise sources are changing constantly due to switching. Therefore, the expression of noise power becomes more complicated.

For simplicity, we assume the noise on the sampling capacitor is still white and the total noise power remains \( kT/C \), the single-side noise power-spectral density (PSD) in the first Nyquist band is written as [10]
\[ \tilde{V}_{noise}^2(f) = \frac{2kT}{Cf_s}. \]  
(10)
This noise PSD is inversely proportional to the sampling frequency. Therefore, a direct RF sampling mixer uses a high sampling frequency (\( N = 1 \)) to reduce the noise PSD in the first Nyquist band. However, this results in a high symbol rate, which is very challenging for the ADC and the backend digital circuits. Therefore, decimation is performed before the sampled signals are passed to the ADC. Unfortunately decimation inevitably folds back wide-band noise into baseband. The direct RF sampling mixer includes embedded notch filter at the frequencies that will alias into the baseband. Since the notch filters suppress wide-band noise at those frequencies before they alias into baseband, the baseband noise PSD does not increase significantly after decimation.

The embedded noise-suppressing filter is constructed as shown in Fig. 2. The history capacitor \( C_{HRS} \) is never reset; this gives a high voltage gain. There should be at least two rotating capacitors \( C_R \). When one is sharing electrical charge with the history capacitor, the other(s) is sharing charge with the next stage circuits or it is reset. Before a rotating capacitor is connected to the history capacitor, it must be reset.

In the direct RF sampling mixer, the number of accumulated samples is equal to the decimation factor \( N \). Assume just before a rotating capacitor is connected, the voltage on the history capacitor is \( y(n - N) \). When there is enough time for complete charge sharing, this voltage drops to
\[ \frac{C_H}{C_H + C_R} y(n - N). \]  
(11)
The voltage added to \( C_H \) and \( C_R \) by the input current is
\[ \frac{1}{C_H + C_R} \sum_{k=0}^{N-1} \int_{(N-k)T_s-T_{on}}^{(N-k)T_s} G_m V_i(t) dt = \sum_{k=0}^{N-1} x(n - k). \]  
(12)
Based on the assumption of complete charge sharing, the rotating capacitor and the history capacitor have the same voltage \( y(n) \) when they separate at the instant \( nT_s \)
\[ y(n) = \frac{C_H}{C_H + C_R} y(n - N) + \sum_{k=0}^{N-1} x(n - k). \]  
(13)
Equation (13) approximately holds for all \( n \) if the history capacitor is much bigger than the rotating capacitors. Applying \( z \) transform to it, the filter transfer function is derived as

\[
H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - \left[\frac{C_H}{C_H + C_R}\right] \cdot z^{-1}} \cdot \frac{1 - z^{-N}}{1 - z^{-1}}. \tag{14}
\]

This expression follows those previously published, and represents a FIR filter and an IIR filter. When the frequency is very small compared with the sampling frequency, \( z \) is estimated as

\[
z^{-N} = e^{-j\omega N T_s} \approx 1 - j N T_s \omega. \tag{15}
\]

Using the frequency-domain approximation, the IIR filter has properties of a single-pole low pass filter, which helps channel selection. In the direct RF sampling mixer, the decimation is performed in two or more stages in order to alleviate the influence of leakage in the capacitors. When the symbol rate is low enough, an ADC is introduced and more sophisticated DSP algorithms can be applied.

IV. INCOMPLETE CHARGE SHARING

One effect that was not previously fully addressed of a direct RF sampling mixer is incomplete charge sharing. When deriving the embedded filtering response, the MOS switch is assumed to have very small drain–source resistance \( (R_{\text{on}}) \) when it is turned on and the charge sharing between the history capacitor and the rotating capacitors takes almost no time to complete. This is generally not true.

Fig. 3 shows the incomplete charge sharing between \( C_H \) and \( C_R \). When the drain–source resistance is not negligible or the rotating capacitor does not have a very small value, it takes more time for the rotating capacitor to take charge from the history capacitor. Solving circuits shown in Fig. 2 leads to

\[
X_1(f) = \frac{1 + j 2\pi f R_{\text{on}} C_R}{j 2\pi f (C_H + C_R + j 2\pi f R_{\text{on}})} G_m h(f) V_i(f) \tag{16}
\]

\[
X_2(f) = \frac{j 2\pi f C_H R_{\text{on}} h(f) G_m}{j 2\pi f C_H C_R} V_i(f) \tag{17}
\]

\[
h(f) = 1 - e^{-j 2\pi f R_{\text{on}}} \tag{18}
\]

where \( X_1(f) \) and \( X_2(f) \) are the voltage difference against the previous sample on \( C_H \) and \( C_R \), respectively. Because of incomplete charge sharing, the voltages on \( C_H \) and \( C_R \) are

\[
y(n) = a y(n - N) + \sum_{k=0}^{N-1} \alpha x_1(n) \tag{19}
\]

\[
u(n) = b y(n - N) + \sum_{k=0}^{N-1} \beta x_2(n) \tag{20}
\]

\[
a = \frac{C_H}{C_H + C_R} \left( 1 + \frac{C_R}{C_H} e^{-j((N-1)T_\Sigma + T_{\text{on}})} / R_{\text{on}} / C_{\text{eqi}} \right) \tag{21}
\]

\[
b = \frac{C_H}{C_H + C_R} \left( 1 - e^{-j((N-1)T_\Sigma + T_{\text{on}})} / R_{\text{on}} / C_{\text{eqi}} \right) \tag{22}
\]

where \( C_{\text{eqi}} \) is the equivalent capacitance when \( C_H \) and \( C_R \) are serial, and \( \alpha \) and \( \beta \) are factors due to capacitor leakage. Now the relationship between input and sampled output becomes

\[
u(z) = \frac{1 - (\alpha z)^{-N}}{1 - (\alpha z)^{-1} x_2(z) + \frac{b z^{-N} (1 - (\beta z)^{-N})}{(1 - (\beta z)^{-1})(1 - a z^{-N})} x_1(z)} \tag{23}
\]

where \( x_1(z) \) and \( x_2(z) \) are given by (16) and (17), respectively. The incomplete charge sharing not only reduces the voltage gain, but also introduces nonlinearity. Seen from (16), (17), (21), and (22), the filter coefficients \( a \) and \( b \) as well as inputs \( x_1(z) \) and \( x_2(z) \) become dependent on the nonlinear drain–source resistance \( R_{\text{on}} \) if the \( RC \) factor is large. Therefore, small \( R_{\text{on}} \) and small \( C_{\text{eqi}} \) to ensure complete charge sharing are important for the system performance.

Fig. 4 shows the comparison of gains calculated with (14), (23) and the transient response by solving the circuits in the time domain

\[
I_0(t) = G_m v_i(t) = (C_H + C_R) V'_R(t) + V_H(t) R_{\text{on}} C_H C_R. \tag{24}
\]

When the \( RC \) factor is small, the assumption of complete charge sharing between \( C_H \) and \( C_R \) is valid. The gains calculated with different methods are similar. In the case of incomplete charge sharing, the modified transfer function will still be accurate, while the transfer function given in references deviates from actual results.
V. ESTIMATION OF THE NOISE FIGURE

In a sampling system, noise level increases due to folding-back of wide-band noise from the source even if there is presumably no additive noise. The wide-band noise from the source is treated as an equivalent resistor \( R \). If the input signal has a PSD \( S(f) \), the double-side signal-to-noise ratio (SNR) of the source defined as

\[
\text{SNR}_1(f) = \frac{S(f)}{2kTR}\tag{25}
\]

The sampling process moves the signal spectra to the baseband with attenuation and also folds back the wide-band noise to the baseband. Because of the embedded notch filter, in the frequency band close to dc, the double-side PSD of the folding-back noise can be written as

\[
V_{\text{noise}}^2(f) \approx 2kTR \sum_{k=-\infty}^{\infty} |g(0)|^2 \sin^2 \left( \frac{k\pi f}{2} \right) = 4kTR|g(0)|^2, \tag{26}
\]

where \( g(0) \) is the system transfer function at dc. The signal PSD at the first Nyquist band is dependent on the distribution of the input signal. In the transmitter side, data are usually interleaved and scrambled. Therefore, the received signal is almost a wide sense stationary sequence. Therefore, in the narrow band close to dc, the signal PSD can be written as

\[
V_{\text{signal}}^2(f) \approx \frac{4}{\pi^2} |g(0)|^2 \left[ S(f + f_0) + S(f - f_0) \right]. \tag{27}
\]

If the signal has a symmetrical PSD, the noise figure in a narrow band around dc is

\[
F = 10\log_{10} \left( \frac{\text{SNR}_{\text{in}}}{\text{SNR}_{\text{out}}} \right) = 10\log_{10} \left( \frac{\pi^2}{4} \right) = 3.9 \text{ dB}, \tag{28}
\]

This matches the minimum noise figure mentioned in [11]. However, if the low-frequency noises have been greatly suppressed by a pre-filter, the noise figure due to wide-band noise folding-back could be as low as 0.9 dB. A typical noise figure of 2.0 dB has been measured for the analog front end of Bluetooth and GSM receivers [5].

Equivalent circuits of the additive thermal noises are shown in Fig. 5. Although there are many thermal noise sources, unlike the noise from the input signal, they do not have a voltage gain larger than one, because they can also be treated as voltage sources. They behave approximately like in conventional track-and-hold circuits that are composed of a voltage source \( E(t) \), a switch and a capacitor \( C \). The noise PSD \( \eta_0 \) of the sampled output signal is

\[
\eta_0(\omega) = \sum_{k=-\infty}^{\infty} |H(\omega - k\omega_0)|^2 \cdot \eta_0(\omega - k\omega_0), \quad \frac{\omega_0}{2} \leq \omega \leq \frac{\omega_0}{2}, \tag{29}
\]

where \( \eta_0 \) is the PSD of the input noise, \( \omega_0 \) is the sampling angular frequency, and \( H(\omega) = Y(\omega)/E(\omega) \) is the system transfer function, which can be derived through a deterministic input \( E(\omega) \) and the sampled output \( Y(\omega) \). \( X_1(\omega) \) and \( X_2(\omega) \) used below are intermediate variables without physical interpretation

\[
Y(\omega) = \frac{X_1(\omega) + \alpha X_2(\omega)}{1 - a e^{-j\omega T}}, \tag{30}
\]

Fig. 5. Equivalent circuits for the first stage of a direct RF sampling mixer. The values of resistors and noise sources are changing due to switching.

Fig. 6. PSD of the sampled noise on \( C_B \). Only RA1 is considered. \( R_{\text{on}}, R_{\text{off}}, C_B \) and \( C_P \) are assumed to be 1 k\( \Omega \), 2 M\( \Omega \), 20 and 0.68 pf, respectively. Total noise means adding all folding back together.

\[
X_1(\omega) = e^{j\omega T} - a e^{-j\omega T} \tag{31}
\]

\[
X_2(\omega) = \frac{1 + j\omega RC}{1 + j\omega RC} \tag{32}
\]

\[
a = e^{-(t_2 - t_1)/RC} \tag{33}
\]

\[
b = e^{-(T - t_2 + t_1)/RC} \tag{34}
\]

where \( E_{\text{on}}(\omega) \) is the input voltage when the switch is on, and \( E_{\text{off}}(\omega) \) is the input voltage when the switch is off. \( T \) is the clock period. From \( t_1 \) to \( t_2 \), the switch is turned on. The voltage on the capacitor is sampled at \( t_1 + t_2 \). If the capacitor is reset in every \( N \) clock periods, the sampled output signal at the \( N \)th period has a spectrum as

\[
Y(\omega) = (1 - a e^{-j\omega T})^N X_1(\omega) + (1 - a e^{-j\omega T})^{N-1} X_2(\omega), \tag{35}
\]

To analyze the transfer functions of thermal noises, the source \( E_{\text{on}}(\omega) \) and \( E_{\text{off}}(\omega) \) in (31) and (32) are treated as uncorrelated independent sources. The input noise PSD in (29) is \( 2kTR_{\text{on}} \) and \( 2kTR_{\text{off}} \) for the on-state and off-state, respectively.

Seen from Fig. 6, the on-state thermal noise of a switch dominates its contribution to PSD of the sampled noise on the sampling capacitor when \( R_S \) is small, which can explain why only the on-state noise is considered in [13]. Although when the MOS switch is off, it has a larger resistance and hence a larger noise power, it cannot establish a large noise voltage on the capacitor because of the leakage in the on-state. As shown in Fig. 5 the internal resistance (\( R_S \)) of the transconductor is always serial with the switches, which makes it difficult to leak out thermal noise, thus the off-state thermal noise becomes dominant. The thermal noise is also strongly shaped by the embedded filters, thus \( 4kTR_{\text{on}} \) is a more accurate estimation of noise PSD than \( kT/C/f_s \). Periodical resetting reduces much of the noise power on the sampling capacitor by reducing the noise power accumulation on the sampling capacitor.
VI. NONLINEARITY DUE TO LARGE GAIN

A large gain is good for the output SNR for the simple reason that the ratio of signal to thermal noise due to drain–source resistance is [12]

\[
\text{SNR} \propto \frac{C \cdot V_{pp}^2}{kT}
\]

(36)

where \(V_{pp}\) is the peak-to-peak value of the signal voltage. For a given input, the more gain, the larger \(V_{pp}\). However, very large gain may force the MOS switch to work in the saturation region. Because the internal resistance of the transistor is not infinitely large, the charging process can be approximately expressed as

\[
V(t) = \frac{1}{C_H + C_R} \int_{t-T_s}^{t} \frac{R_s}{R_s + R_{DS}} I_D(\tau) d\tau
\]

(37)

where \(R_s\) is the output resistance of the transistor, \(R_{DS}\) is the drain–source resistance of the MOS transistor. In the saturation region \(R_{DS}\) becomes comparable with \(R_s\), and it is strongly dependent on the drain–source current. Therefore, nonlinearity becomes severe when gain is very large. The maximum voltage on the sampling capacitor is calculated as

\[
\text{Max}[V(n)] |_{R_{DS} \ll R_s} \leq \frac{N}{V_H - V_{ths}} \left[ \frac{A G_m}{A C_R} \right] \left[ 1 + \frac{C_H}{C_R} \right]
\]

(38)

where \(V_H\) is the high voltage of the clock signal, \(A\) is the amplitude of the input voltage, \(V_{ths}\) is the threshold voltage of the MOS transistor, and \(N\) is the decimation factor. With the assumption that \(C_R\) is much smaller than \(C_H\), the maximum voltage becomes independent with \(C_H\). For the second stage decimation and charge sharing, the transfer function is

\[
H_2(z) = \frac{1}{1 - \frac{C_R}{C_B + C_R} z^{-1}} \left[ \frac{1 - z^{-N_2}}{1 - z^{-1}} \right] C_R \quad \text{where} \quad C_H, C_R, C_B
\]

(39)

where \(C_H\) is the capacitance of the bank capacitor which samples the charge on the \(N_2\) rotating capacitors. The maximum voltage gain of the second stage is one. Low distortion design of the first stage remains critical in the overall design.

Although the current source effectively reduces influence of the nonlinear drain–source resistance of the sampling MOS switch, the influence of the nonlinear drain–source resistance of the switch connecting the history capacitor and the rotating capacitor is not reduced by the current source. Seen from (21) and (22), its influence can be minimized by a very small \(RC\) factor. Large voltage gain also increases the drain–source resistance and deteriorates the system linearity as shown in Fig. 7.

A smaller capacitor or a MOS switch with smaller on-state resistance can lead to a small \(RC\) factor which will reduce the influence of nonlinear drain–source resistance. However, either case makes it more susceptible to clock feed-through unless actions are taken to prevent it.

VII. SUMMARY AND CONCLUSION

An analytical model representing nonlinearity in a direct RF sampling mixer is proposed. Design considerations such as incomplete charge transfer, noise figure estimation, and gain against nonlinearity are addressed. Insights proposed through this work can also be used to analyze other windowed integration samplers and subsamplers that have become important for SDR and ADC applications.

REFERENCES