RF Systems Course: RF Concepts I

Introduction

- In the next two lectures some general issues in RF design will be considered.
- Nonidealities including noise and linearity
- ideal circuit, ex. ideal amplifier, produces a perfect copy of the input signal
- real circuit adds noise/distortion
- Noise limits min. detectable signal
- nonlinearities limit max signal amplitude
- specifications for linearity, noise etc. must be determined before the circuit can be designed.
- In order to determine specs, impact of noise/distortion need to be understood.

Gain

- blocks designed to sense some property of the input signal (the input voltage, current, or power) -> amplify this or some other property at the output.

- many different types of gain can be defined,

- ex. Power gain:

$$G = \frac{P_{\text{out}}}{P_{\text{in}}}$$

Type of Gain	*Equation*	*Description*
Power gain	$G = 10\log\left(\frac{P_{out}}{P_{in}}\right)$	The ratio of ouptut power to input power.
Voltage gain	$A_v = 20\log\left(\frac{V_{out}}{V_{in}}\right)$	The ratio of the output voltage to the input voltage.
Current gain	$A_i = 20\log\left(\frac{i_{out}}{i_{in}}\right)$	The ratio of the output current to the input current.
Maximum available power gain	$G_A = 10\log\left(\frac{P_{out\_av}}{P_{in\_av}}\right)$	The ratio of maximum available output power to the maximum available input power.
Transducer power gain	$G_T = 10\log\left(\frac{P_{out}}{P_{in\_av}}\right)$	The ratio of the output power to the maximum available input power.
Open ciruit voltage gain	$A_{v\_oc} = 20\log\left(\frac{v_{out\_OC}}{v_{in}}\right)$	The ratio of the output voltage to the input voltage when the circuit is loaded with only an open circuit load.

Noise

- Signal detection more difficult in presence of noise.
- In addition to the desired signal, the receiver is also picking up noise from the rest of the universe.
- thermal energy moves atoms and electrons around in a random way, leading to random currents, seen as noise.
- Noise can also come from microwave ovens, cell phones, pagers, radio antennas etc.
- RF designers mostly concerned with how much noise is being added by the circuits in the transceiver.
- At input, some noise power present which defines the noise floor.
- Min. detectable signal must be higher than the noise floor by some SNR to detect signals reliably and to compensate for additional noise added by circuitry.
- To find the total noise due to a number of sources, relationship of sources with each other has to be considered.
- most common assumption is that all noise sources are random, have no relationship with each other ->uncorrelated
- In this case noise power is added instead of noise voltage.
- noise at different frequencies is uncorrelated, noise power is added.
- if two sources are correlated, the voltages can be added.
- E.g. correlated noise is seen at the outputs of two separate paths that have the same origin.

Thermal Noise

- Noise in resistors is generated by thermal energy causing random electron motion. The thermal noise spectral density in a resistor is given by:

$$N_{\text{resistor}} = 4kTR$$

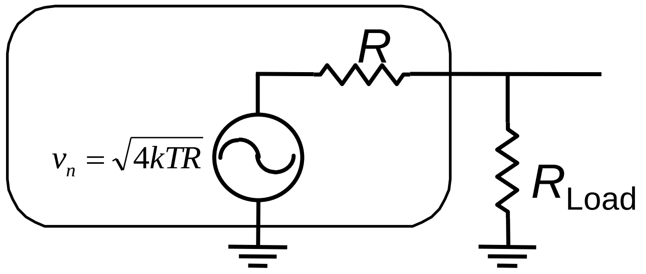
- where T is the temperature in Kelvin of the resistor, k is Boltzmann's constant (1.38 x 10^{-23} Joules/K) and R is the value of the resistor.
- Noise power spectral density has the units of V^2/Hz.
- Power a resistor produces in a finite bandwidth:

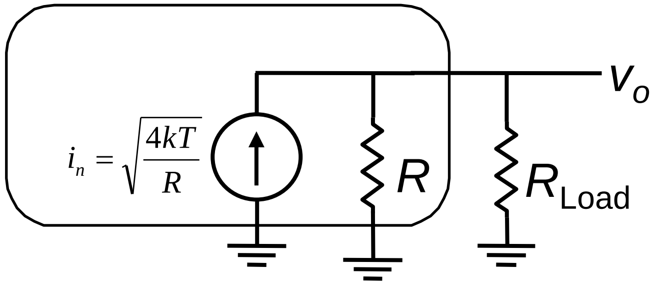
$$v_n^{\,2} = 4kTR\Delta f$$

- where v_n is the rms value of the noise voltage in the bandwidth Δf.
- as a noise current :

$$i_n^{\,2} = \frac{4kT\Delta f}{R}$$

- Thermal noise is white (has a constant PSD)





Available Noise Power

- Max power is transferred to load when R_{LOAD} is equal to R.
- Then v_o is equal to $v_n/2$.
- The output PSD P_o is :

$$P_o = \frac{v_o^2}{R} = \frac{v_n^2}{4R} = kT$$

- Available power is kT, independent of resistor size.
- kT is in Watts/Hz
- Total power P_{out} in Watts:

$$P_{\text{out}} = kTB$$

- noise power from antenna with input resistance R equivalent to resistor with the same value.
- available noise power from an antenna:

$$P_{\text{available}} = kT = 4 \cdot 10^{-21}\,\text{W/Hz}$$

$$P_{\text{available}} = 10\log_{10}\left(\frac{4 \times 10^{-21}}{1 \times 10^{-3}}\right) = -174\,\text{dBm/Hz}$$

at T = 290K,

- Using 290K is appropriate where the antenna is pointed at the horizon.
- if the antenna were pointed at the sky, the equivalent noise temperature would be much lower, more typically 50K.

Signal-To-Noise Ratio

- noise floor depends on bandwidth.
- E.g. bandwidth of 200 kHz:

$$\text{Noise Floor} = kTB = 4\times10^{-21}\times200000 = 8\times10^{-16}W$$

$$\text{Noise Floor} = -174\text{dBm/Hz} + 10\log_{10}(200{,}000) = -121\text{dBm}$$

$$\text{SNR} = \frac{S}{\text{Noise Floor}}$$

- if electronics added no noise and detector requires SNR of 0 dB, then Min signal of -121 dBm can be detected.
- Min. detectable signal in a receiver called receiver sensitivity.
- required SNR depends on a variety of factors, such as modulation scheme, bit rate, energy per bit, IF filter bandwidth, detection method (for example, synchronous or not), interference levels, etc.
- for a bit error rate of 10^{-3} need 7dB for QPSK, 12 dB for 16 QAM, 17 dB for 64 QAM
- lower BER often required (for example, 10^{-6}) need higher SNR

Noise Figure

- Noise added by electronics will directly add to the noise from the input.
- min detectable signal level must be modified to include the noise from circuitry.
- Noise from electronics described by Noise Factor F

$$F = \frac{\mathrm{SNR}_i}{\mathrm{SNR}_o} = \frac{S_i / N_{i(\mathrm{source})}}{S_o / N_{o(\mathrm{total})}} = \frac{S_i / N_{i(\mathrm{source})}}{(S_i \cdot G) / N_{o(\mathrm{total})}} = \frac{N_{o(\mathrm{total})}}{G \cdot N_{i(\mathrm{source})}}$$

$$S_o = G \cdot S_i$$

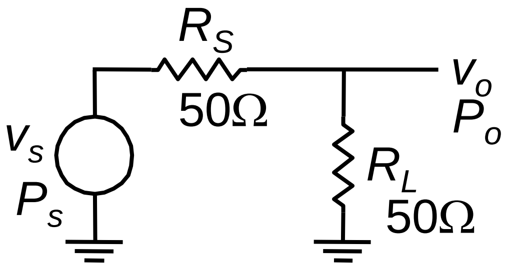
$$N_{o(\mathrm{total})} = N_{o(\mathrm{source})} + N_{o(\mathrm{added})}$$

$$F = \frac{N_{o(\mathrm{total})}}{G \cdot N_{i(\mathrm{source})}} = \frac{N_{o(\mathrm{total})}}{N_{o(\mathrm{source})}} = \frac{N_{o(\mathrm{source})} + N_{o(\mathrm{added})}}{N_{o(\mathrm{source})}} = 1 + \frac{N_{o(\mathrm{added})}}{N_{o(\mathrm{source})}}$$

$$\mathrm{NF} = 10\, log_{10}\, F$$

- In Rx for components with loss (such as switches and filters), noise figure = attenuation.
- E.g. filter with 3 dB of loss has NF= 3 dB.
- output noise ≅ input noise, but signal attenuated by 3 dB -> degradation of SNR by 3 dB.

Example Noise Calculation



Determine noise voltage per unit bandwidth at output.
find the noise factor, assuming that R_L does not contribute to the noise factor, and compare to the case where R_L does contribute to the noise factor.

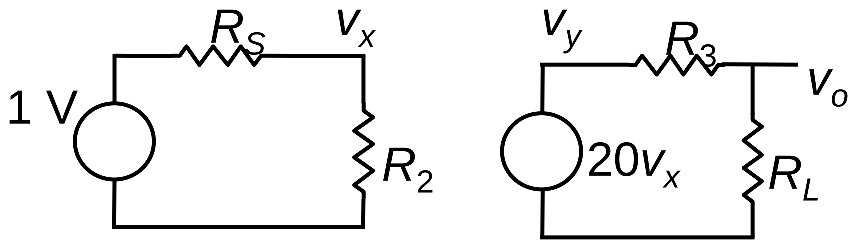
Solution:
The rms noise voltage from the 50Ω source is $\sqrt{4kTR} = 0.894\ \text{nV}/\sqrt{\text{Hz}}$ at a temperature of 290K, which, after the voltage divider, becomes one-half of this value or $v_o = 0.447\ \text{nV}/\sqrt{\text{Hz}}$.

The complete available power from the source is delivered to the load. In this case,

$$P_o = P_{\text{in(available)}} = kT = 4 \cdot 10^{-21}$$

At the output, the complete noise power (available) appears and so, if R_L is noiseless, the noise factor is 1. However, if R_L has noise of $\sqrt{4kTR_L}\ \text{V}/\sqrt{\text{Hz}}$, then at the output, the total noise power is $2kT$ where kT is from R_S and kT is from R_L. Therefore, for a resistively matched circuit, the noise figure is 3 dB. Note that the output noise voltage is $0.45\ \text{nV}/\sqrt{\text{Hz}}$ from each resistor for a total of $\sqrt{2} \times 0.45\ \text{nV}/\sqrt{\text{Hz}} = 0.636\ \text{nV}/\sqrt{\text{Hz}}$ (with noise, the power adds because the noise voltage is uncorrelated).

Example Noise Calculation



All R= 50Ω.
noise at the output due to all resistors?
determine the circuit noise figure and SNR assuming 1 MHz bandwidth and the input is a 1 V sine wave.

In this example, at v_x the noise is still due to only R_S and R_2. As in the previous example, the noise at this point is 0.636 nV/$\sqrt{\text{Hz}}$. The signal at this point is 0.5V, thus at point v_y, the signal is 10V and the noise due to the two input resistors R_S and R_2 is $0.636 \times 20 = 12.72$ nV/$\sqrt{\text{Hz}}$. At the output, the signal and noise from the input sources, as well as the noise from the two output resistors, all see a voltage divider. Thus, one can calculate the individual components. For the combination of R_S and R_2, one obtains

$$v_{R_S+R_2} = 0.5 \times 12.72\ \text{nV}/\sqrt{\text{Hz}} = 6.36\ \text{nV}/\sqrt{\text{Hz}}$$

The noise from the source can be determined from this equation:

$$v_{R_S} = \frac{6.36\ \text{nV}/\sqrt{\text{Hz}}}{\sqrt{2}} = 4.5\ \text{nV}/\sqrt{\text{Hz}}$$

For the other resistors, the voltage is

$$v_{R_3} = 0.5 \times 0.9\text{nV}/\sqrt{\text{Hz}} = 0.45\text{nV}/\sqrt{\text{Hz}}$$

$$v_{R_L} = 0.5 \times 0.9\text{nV}/\sqrt{\text{Hz}} = 0.45\text{nV}/\sqrt{\text{Hz}}$$

Total output noise is given by

$$v_{\text{no(total)}} = \sqrt{v_{R_S+R_2}^2 + v_{R_3}^2 + v_{R_L}^2}$$

$$= \sqrt{6.36^2 + 0.45^2 + 0.45^2}\left(\text{nV}/\sqrt{\text{Hz}}\right) = 6.392\ \text{nV}/\sqrt{\text{Hz}}$$

Therefore, the noise figure can now be determined:

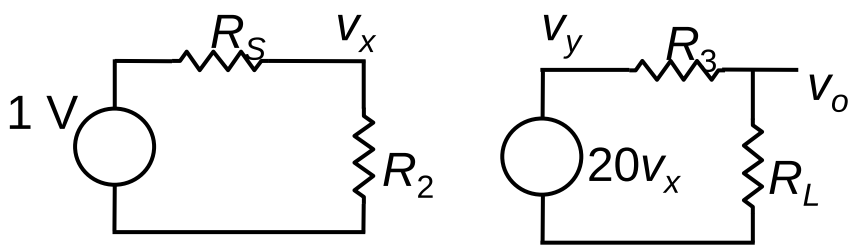
$$NF = 10\log F = 10\log\left(\frac{N_{o(\text{total})}}{N_{o(\text{source})}}\right) = 10\log\left(\frac{6.392}{4.5}\right)^2 = 10\log(1.417)^2 = 3.03\ \text{dB}$$

Since the output voltage also sees a voltage divider of 1/2, it has a value of 5V. Thus, the SNR is

$$SNR = 20\log\left(\frac{5}{\frac{6.392\ \text{nV}}{\sqrt{\text{Hz}}}\cdot\sqrt{1\ \text{MHz}}}\right) = 117.9\ \text{dB}$$

This example illustrates that noise from the source and amplifier input resistance are the dominant noise sources in the circuit. Each resistor at the input provided 4.5 nV/$\sqrt{\text{Hz}}$, while the two resistors behind the amplifier each only contribute 0.45 nV/$\sqrt{\text{Hz}}$. Thus, as explained earlier, after a gain stage, noise is less important.

Example Noise Calculation



Find the noise figure again, but now assume that R_2 = 500Ω.

As before, the output noise due to the resistors is as follows:

$$v_{no(R_s)} = \left(0.9 \times \frac{500}{550} \times 20 \times 0.5\right) \text{ nV}/\sqrt{\text{Hz}} = 8.181 \text{ nV}/\sqrt{\text{Hz}}$$

where 500/550 accounts for the voltage division from the noise source to the node v_x.

$$v_{no(R_2)} = \left(0.9 \times \sqrt{10} \times \frac{50}{550} \times 20 \times 0.5\right) \text{ nV}/\sqrt{\text{Hz}} = 2.587 \text{ nV}/\sqrt{\text{Hz}}$$

where the $\sqrt{10}$ accounts for the higher noise in a 500Ω resistor compared to a 50Ω resistor.

$$v_{no(R_3)} = (0.9 \times 0.5) \text{ nV}/\sqrt{\text{Hz}} = 0.45 \text{ nV}/\sqrt{\text{Hz}}$$

$$v_{no(R_L)} = (0.9 \times 0.5) \text{ nV}/\sqrt{\text{Hz}} = 0.45 \text{ nV}/\sqrt{\text{Hz}}$$

The total output noise voltage is

$$v_{no(total)} = \sqrt{v_{R_S}^2 + v_{R_2}^2 + v_{R_3}^2 + v_{R_L}^2} = \left(\sqrt{8.181^2 + 2.587^2 + 0.45^2 + 0.45^2}\right) \text{nV}/\sqrt{\text{Hz}}$$

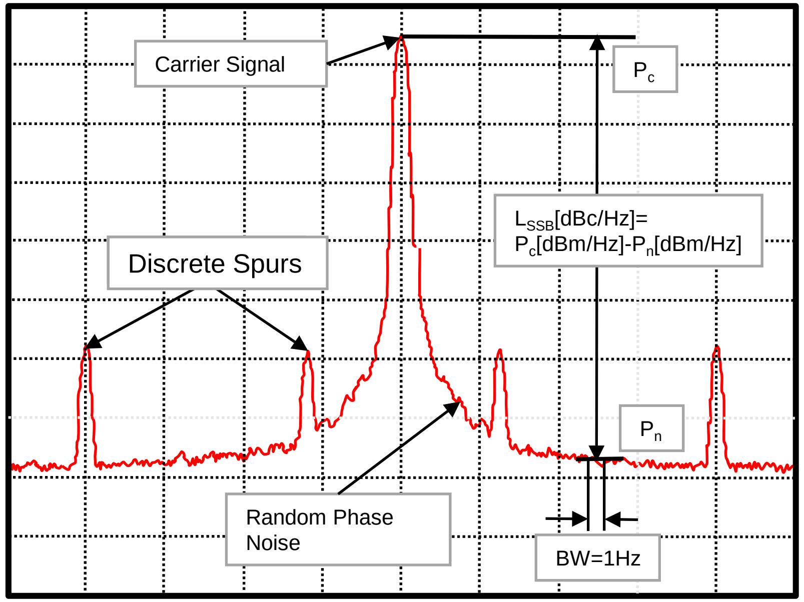
$$= 8.604 \text{ nV}/\sqrt{\text{Hz}}$$

The noise figure is

$$NF = 10\log\left(\frac{N_{o(total)}}{N_{o(source)}}\right) = 10\log\left(\frac{8.604}{8.181}\right)^2 = 0.438 \text{ dB}$$

- Note: this circuit is unmatched at the input.
- E.g. shows mismatched circuit may have better NF
- assumes possible to build voltage amp that requires little power at the input.
- This may be possible on an IC.
- if transmission lines are included, power transfer will suffer.
- matching circuit may need to be added.

Phase Noise

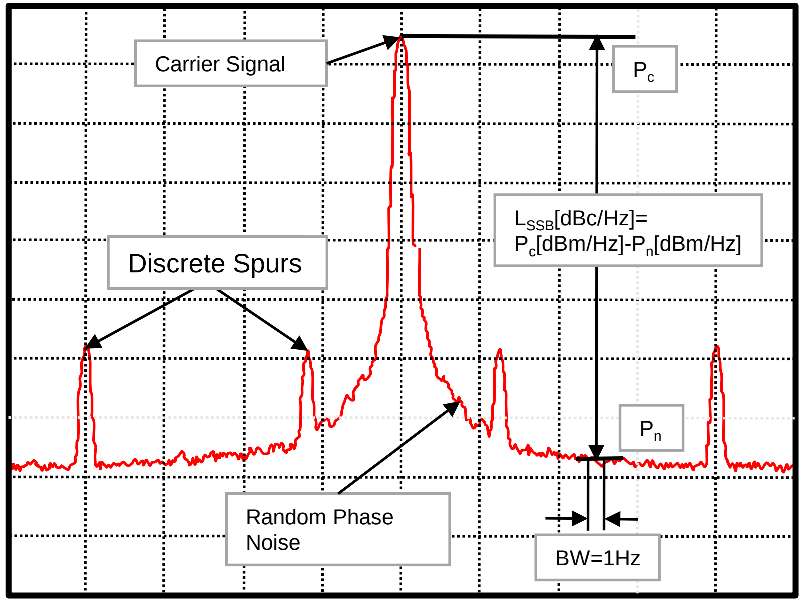


- Radios use reference tones to perform frequency conversion.
- should be perfect ->have energy only at desired frequency.
- LO noise performance (phase noise) measures how real signal diverges from an ideal impulse function
- primarily concerned with noise that causes fluctuations in phase rather than amplitude fluctuations, since the output typically has a fixed, limited amplitude.

$$v_{out}(t) = V_o \cos(\omega_{LO} t + \varphi_n(t))$$

- PN units dBc/Hz
- timing jitter rad^2/Hz
- phase fluctuation may be random noise or discrete spurious tones

Phase Noise



Assume the phase fluctuation is of a sinusoidal form as:

$$\varphi_n(t) = \varphi_p \sin(\omega_m t)$$

$$v_{\text{out}}(t) = V_0 \cos[\omega_{\text{LO}} t + \varphi_p \sin(\omega_m t)] = V_0 [\cos(\omega_{\text{LO}} t)\cos(\varphi_p \sin(\omega_m t)) - \sin(\omega_{\text{LO}} t)\sin(\varphi_p \sin(\omega_m t))]$$

For a small phase fluctuation

$$v_0(t) = V_0 [\cos(\omega_{\text{LO}} t) - \varphi_p \sin(\omega_m t)\sin(\omega_{\text{LO}} t)]$$

$$= V_0 \left[\cos(\omega_{\text{LO}} t) - \frac{\varphi_p}{2} [\cos([\omega_{\text{LO}} - \omega_m]t) - \cos([\omega_{\text{LO}} + \omega_m]t)] \right]$$

- phase-modulated signal includes carrier signal tone, 2 sidebands at offset frequency,\
- SA measures phase-noise power in dBm/Hz, but often phase noise is reported relative to the carrier power as:

$$\varphi_n^{\,2}(\Delta\omega) = \frac{Noise(\omega_{\text{LO}} + \Delta\omega)}{P_{\text{carrier}}(\omega_{\text{LO}})}$$

Phase Noise

- Often quoted as so many dBs down from the carrier (dBc/Hz)
- SSB PN defined as ratio of power in one phase modulation sideband per Hertz of bandwidth, at an offset $\Delta\omega$ away from the carrier, to total signal power.
- The SSB PN PSD to carrier ratio, in units of [dBc/Hz], is defined as

$$PN_{SSB}(\Delta\omega) = 10\log\left[\frac{Noise(\omega_{LO}+\Delta\omega)}{P_{carrier}(\omega_{LO})}\right]$$

$$PN_{SSB}(\Delta\omega) = 10\log\left[\frac{\frac{1}{2}\left(\frac{V_0\varphi_p}{2}\right)^2}{\frac{1}{2}V_0^2}\right] = 10\log\left[\frac{\varphi_p^2}{4}\right] = 10\log\left[\frac{\varphi_{rms}^2}{2}\right]$$

DSB PN:

$$PN_{DSB}(\Delta\omega) = 10\log\left[\frac{Noise(\omega_{LO}+\Delta\omega) + Noise(\omega_{LO}-\Delta\omega)}{P_{carrier}(\omega_{LO})}\right] = 10\log\left[\varphi_{rms}^2\right]$$

- From either SSB or DSB PN, rms jitter is:

$$\varphi_{rms}(\Delta f) = \frac{180}{\pi}\sqrt{10^{\frac{PN_{DSB}(\Delta f)}{10}}} = \frac{180\sqrt{2}}{\pi}\sqrt{10^{\frac{PN_{SSB}(\Delta f)}{10}}}\left[\deg/\sqrt{\text{Hz}}\right]$$

- also quite common to quote integrated PN
- rms integrated PN of a synthesizer:

$$\text{IntPN}_{rms} = \sqrt{\int_{\Delta f_1}^{\Delta f_2} \varphi_{rms}{}^2(f)\,df}$$

- dividing or multiplying a signal in freq domain also multiplies or divides the PN:

$$\varphi^2{}_{rms}(N\omega_{LO}+\Delta\omega) = N^2\cdot\varphi^2{}_{rms}(\omega_{LO}+\Delta\omega)$$

$$\varphi^2{}_{rms}\left(\frac{\omega_{LO}}{N}+\Delta\omega\right) = \frac{\varphi^2{}_{rms}(\omega_{LO}+\Delta\omega)}{N^2}$$

- assumes circuit is noiseless.
- PN is scaled by N^2 rather than N to get units of V^2 rather than noise voltage.