ELECTRONICS 2501: Circuits and Signals

The first ‘electrical engineering’ course

Basic engineering circuit analysis

Fundamental and general principals that will form the basis for analysis in all branches of engineering and science

- Electronic circuit design
- Power systems
- Radio and microwave systems
- Heat transfer
- Vibration analysis
- Electrochemistry

Etc…

What will you finish the course with?

- Good understanding of the key electrical qualities voltage, current, and power in ‘passive’ circuits -> resistors, capacitors, and inductors (combined with simple sources)
- While most electronic circuits will require you to learn more, many applications can be studied using techniques from this course. For example, a good portion of power systems analysis is done with RLC circuits.

- Course Outline
- Introduction to signals
    - Definition of signals
    - Signal types
    - Periodic signals
    - Describing periodic signals
    - Examples
- Properties of signals
    - Instantaneous
    - Peak
    - Peak to peak
    - Average
    - RMS
- Examples

Dictionary Definition:

Signal

1. Sign, indication
2. (a) an act, event, or watchword that has been agreed on as the occasion of concerted action, (b) something that indicates action
3. Something (such as a sound, gesture, or object) that conveys notice or warning
4. (a) an object used to transmit or convey information beyond the range of human voice, (b) the sound or image conveyed in telegraphy, telephony, radio, radar, or television, (c) a detectable physical quantity or impulse (such as a voltage, current, or magnetic field strength) by which messages or information can be transmitted.

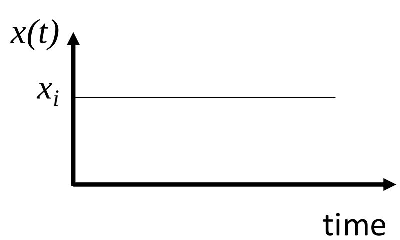
It should be added to 4(c) messages, information or POWER can be transmitted.

<u>Types of Signals</u>

Perhaps the simplest is a constant time-invariant level commonly referred to as a DC (from direct current)

Examples include:

- Battery output
- Wall adapter output



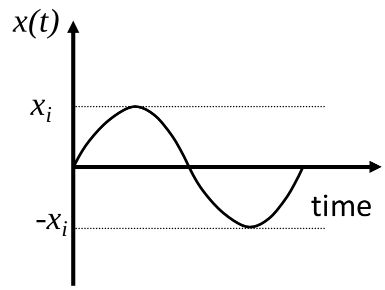
A little more complex to analyze would be a sinusoidally varying signal commonly referred to as AC (from alternating current):

$$x(t) = A \sin(\omega t)$$

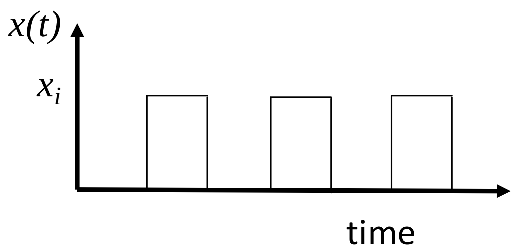
- A is the amplitude
- ω is the frequency is rads/s

Examples include:

- Power distribution system
- Radio carriers

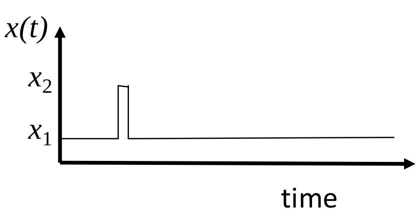


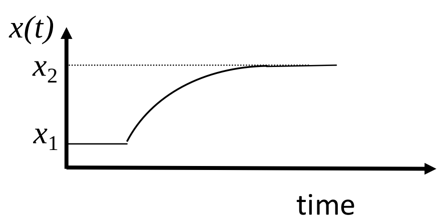
There are other signals that are not sinusoidal but still vary regularly in time - periodic signals. A square wave is one example. e.g. computer clock signal.



And there are many signals that are not periodic

- Pulse - fairly rapid change then return to normal
- Transient - more gradual change over time





We will see all these types of signals starting with DC. We will be able to develop all the analysis techniques we need. However, to extend these to AC signals requires some interesting representations for sinusoidal signals. Finally, we will explore how sinusoidal signals can be combined to represent most other periodic signals.

Periodic Signals

Why use periodic signals? It turns out a sinusoid is fairly easy to generate using a rotating machine. It is also easy to change the level, so power generation and distribution is dominated by sinusoids.

Periodic signals also include both level and time information, so they are very useful for timing digital systems.

Describing a Periodic Signal

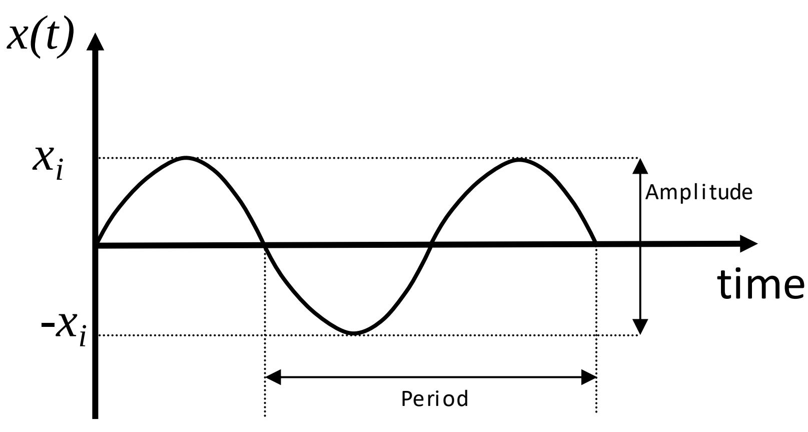
An ideal constant signal has one property -> level or amplitude

To measure this, we can read a single number -> DMM (digital multi meter)

A periodic signal has a shape, a level (amplitude) and a repeat time (period)

To measure this we need an amplitude vs. time -> use an oscilloscope

However, if we already know something about the signal (e.g. power system) we may make do with a single number -> RMS



Periodic Signals

So say we have $x(t) = A\sin(\omega t)$ since *sin* goes from -1 to +1, *x(t)* goes from –A to +A.

sin repeats every 2π radians, so the period of x(t) is: $T = \frac{2\pi}{\omega}$

we often like to think in terms of cycles/sec = Hz rather than radians/sec so:

$\omega = 2\pi f$ and $T = \frac{1}{f}$

Instantaneous value -> value at an instant in time

Peak value -> maximum value.

Peak-to-peak value -> difference between maximum and minimum value

Average value -> $\frac{1}{T_2-T_1}\int_{T_1}^{T_2} f(t)dt$ **<u>This is zero for many periodic signals</u>**

RMS (root mean square) -> $\sqrt{\frac{1}{T_2-T_1}\int_{T_1}^{T_2}(f(t))^2dt}$

One of the most common signals we are interested in is the sinusoid.

For

$$i(t) = I_1 \cos(\omega t - \theta)$$

$$I_{RMS} = \sqrt{\frac{1}{T}\int_0^T {I_1}^2 \cos^2(\omega t - \theta)\, dt}$$

Note that:

$$\cos^2(\phi) = \frac{1}{2}(1 + \cos 2\phi)$$

$$I_{RMS} = I_1\sqrt{\frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} \frac{1}{2}(1 + \cos(2\omega t - 2\theta))dt}$$

Since the cosine averages to zero we are left with:

$$I_{RMS} = I_1\sqrt{\frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} \frac{1}{2}dt}$$

So

$$I_{RMS} = I_1\sqrt{\frac{\omega}{2\pi}\frac{1}{2}\left(\frac{2\pi}{\omega} - 0\right)} = \frac{I_1}{\sqrt{2}}$$

Important as many zero average periodic signals transfer power, which can be indicated by RMS level.

Therefore, RMS is more useful than the average value because is gives us information about the amplitude and power in a waveform which **<u>cannot</u>** be found from the average value.

For example:

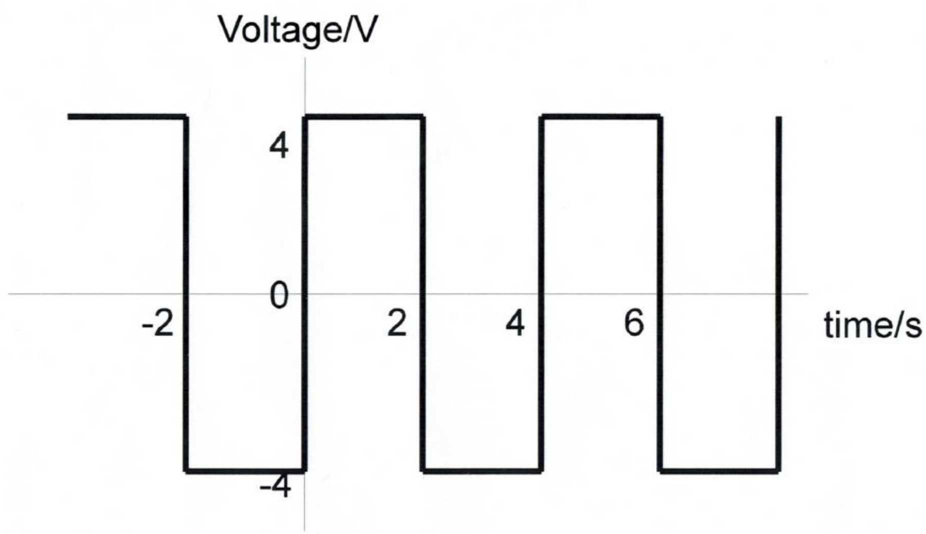
A sinewave with an amplitude of 1 has a zero average value.

A sinewave with an amplitude of 1,000,000 has a zero average value.

A sinewave with an amplitude of 1 has an RMS value of 0.707.

A sinewave with an amplitude of 1,000,000 has an RMS value of 707,000.

Example:



a) What is the frequency of the signal?
b) What is the instantaneous value at 5s?
c) What is the average voltage?
d) What is the V_{RMS}?

SI Units

- Clearly you have familiarity with SI units
- What is interesting about circuit analysis and electrical quantities is that they vary over many orders of magnitude and you will encounter SI prefixes not commonly used in other fields of study.
- Integrated circuit dimensions are measured in nano (10^{-9}) meters, while clock speeds (frequencies) are measured in giga (10^{9}) Hz.
- Component values can be in pico (p), nano (n), micro (μ), milli (m), kilo (k) or mega (M) units. Make sure you know the multipliers or your calculations will be wrong. This is a simple thing to remember, so markers probably won't be sympathetic to these mistakes. ******Also a good engineer should know without picking up a calculator when the answer is off by an order of magnitude, let alone three or more orders of magnitude!!! *********

Basic Quantities

Electric charge is the most elementary quantity we need to deal with. Usually labeled *q.* We are particularly interested in motion of charge, since this results in energy transfer. Situations in which motion is constrained to a closed path (circuit) are of particular interest.

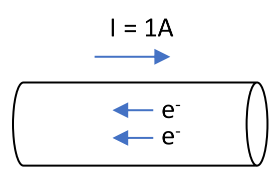
Movement of charge -> time rate of change of charge = current.

If charge is a function of time $q(t)$, current is $i(t) = \frac{dq(t)}{dt}$ or $q(t) = \int_{-\infty}^{t} i(x)dx$

Charge is measured in coulombs

Current is measured in Amperes = coulombs/sec.

Although you are probably aware the common charge 'carrier' is the electron (in metals for example), the convention is to define current flow as the direction of movement of **positive** charge.



1C/s of charge passes left to right.

Note current flows through a component/device so what is the driving force?

Charge motion leads to energy transfer, so at different positions charges have different potential energies or potentials. Also called electromotive force or EMF. In engineering this is most often called voltage.

There are many physical analogies - gravitation -fluid flow

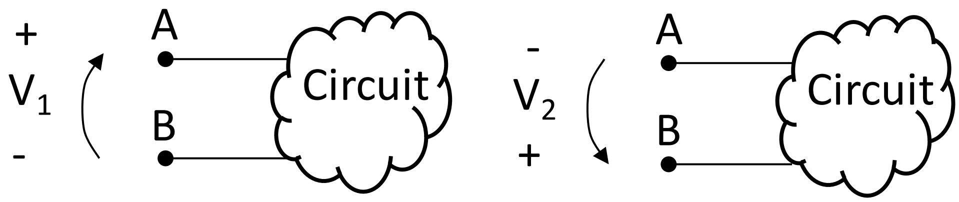
Voltage is a difference in potential energy between two points

For positive charge it is measured across elements.

The sign of the voltage depends on which point is chosen as the reference -> and should be **defined**

voltage is more formally defined as the work/unit charge required to move charge from one location to another, so it is in units of

Volts = Joule (energy) / coulomb (charge) $v = \frac{dw}{dq}$



**** V_1 is defined as the voltage FROM point B TO point A.

**** V2 is defined as the voltage FROM point A TO point B. Thus:

$V_1 = V_A - V_B$ $V_2 = V_B - V_A = -(V_A - V_B)$

Therefore: $V_2 = -V_1$

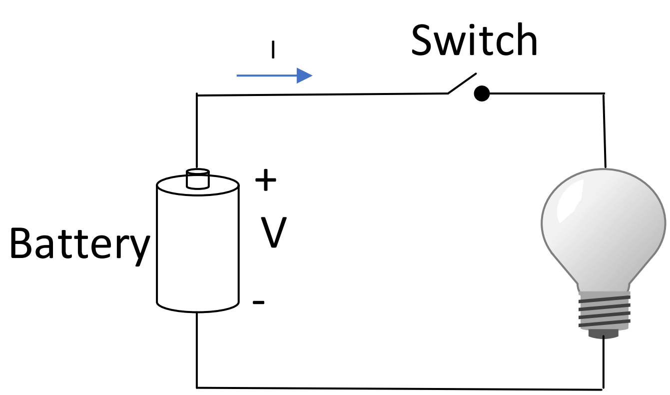
This reveals a problem that confuses students when we start analyzing circuits -> you can choose different references for voltages, but if you are careful and consistent the answer still comes out the same in the end!

In many cases a common reference or ground is used to avoid confusion, errors, and dangerous accidents.

Classic first circuit -> flashlight

A Circuit

Just like a race course, an electrical circuit must form a loop (although it may not always be obvious)



New flashlights use LED's which is only slightly more complicated.

If we close the switch, charge flows through the bulb filament, heating it up and making it glow.

Chemical energy (battery) -> electrical energy -> heat + light

What happens if we add another battery?

Depending how we do it, the light is brighter.

Battery provides energy -> source

Element absorbs (dissipates) energy -> sink and the two need to balance -> conservation.

Look at units $v = \frac{dw}{dq}$ $i = \frac{dq}{dt}$ -> $v \cdot i = \frac{dw}{dq} \cdot \frac{dq}{dt} = \frac{dw}{dt}$

$v \cdot i$ = work/sec = J/s = **POWER**

Power

Since v and i can vary in time, we should also consider p varies in time:

$p(t) = v(t) \cdot i(t)$

And energy W delivered in a time $\Delta t = t_2 - t_1$ is given by:

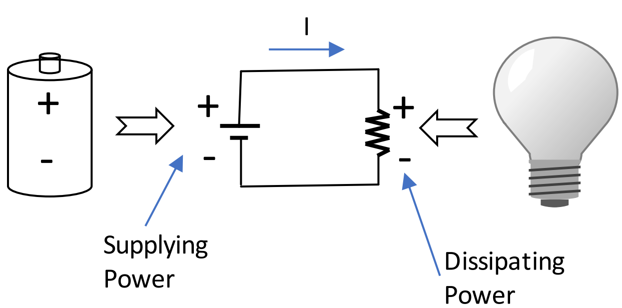
$$W = \int_{t_1}^{t_2} p(t)dt = \int_{t_1}^{t_2} v(t) \cdot i(t)dt$$

We have a sign for v and i. Therefore, p could also take a sign, which can be interpreted as power absorbed (+ive) or power supplied (-ive) by an element.

If current enters the positive voltage terminal of an element -> power absorbed

If current leaves the positive voltage terminal of an element -> power supplied

Consider our flashlight:



Of course conservation laws apply to circuits in particular conservation of charge, energy, and power.

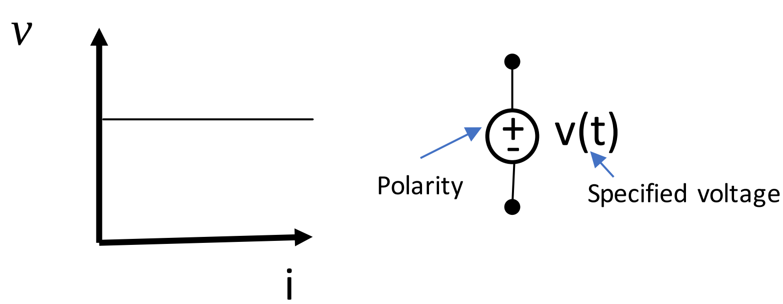
Sum of powers supplied must equal sum of powers dissipated in a circuit. (book refers to Tellegen's Theorem)

Sources – Current and Voltage

We will require power supplied to our circuits, which we will represent with different types of ideal sources. Note that we can represent a real source fairly accurately by an ideal source plus a loss element (resistor).

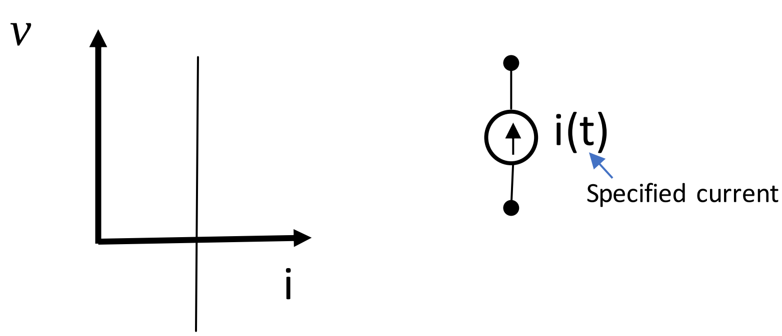
We will find uses for both independent sources which allow us to provide specific values of voltage and current and dependent sources with output that depends on some variable in our circuit.

Independent voltage source maintains a specified voltage between terminals regardless of the current through it.



We have plotted v vs. i which is a common way of characterizing circuit elements. Note the quantities may be time varying.

Independent current source maintains a specified current regardless of the voltage across its terminals.



Independent sources usually supply power to a circuit, but in some situations they may absorb power (e.g. charging a battery).

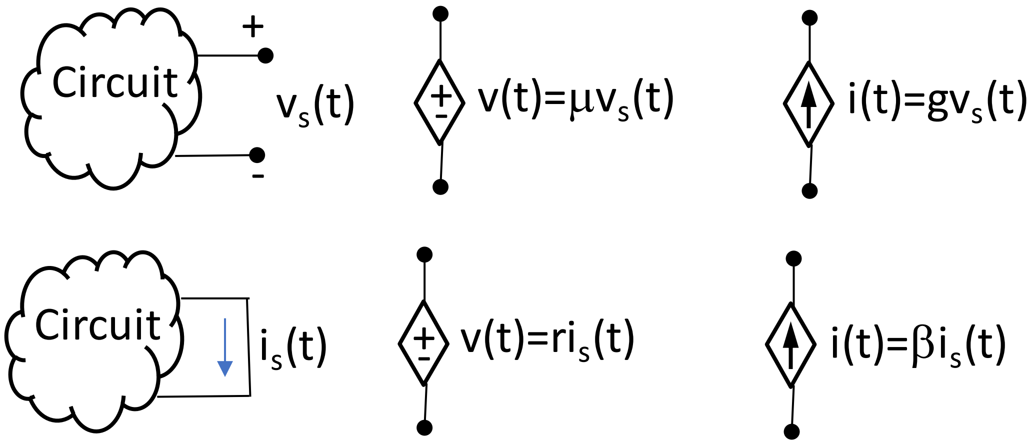
Note real sources cannot provide infinite power, so a voltage source will provide decreasing voltage as current increases, and a current source will provide decreasing current as voltage increases.

We will use ideal independent sources extensively in the course.

Dependent sources provide voltage or current determined by voltage or current at a specified location in the circuit.

You will use these extensively to model more complex electrical components such as transistors.

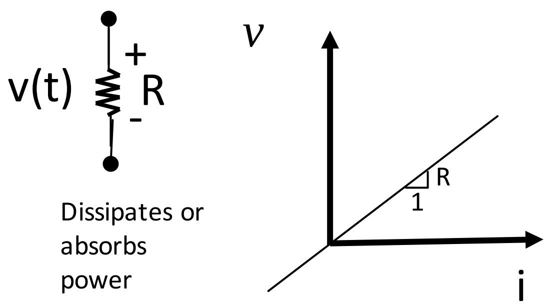
They can be either voltage or current controlled.



OHM's Law

Voltage across a resistance is proportional to the current through it.

A resistive element is called a resistor, R then $v(t) = R\ i(t)$ (R>0) for an ideal resistor which has a constant value for all currents. Actually it will heat up and change depending on the material.



Resistance is measured in Ohms   1Ω = 1V/A

We already know power $p(t) = v(t) \cdot i(t)$

Using ohm's law: $p(t) = v(t) \cdot \left(\frac{v(t)}{R}\right) = \frac{v^2}{R}$

Alternatively: $p(t) = i(t) \cdot R \cdot i(t) = i^2 R$

Sometimes it is useful to use conductance $G = \frac{1}{R}$ in siemens is = 1A/V

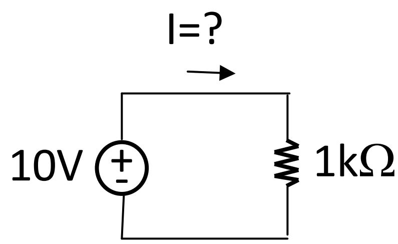
Then $i(t) = G \cdot v(t)$ and $p(t) = \frac{i^2(t)}{G} = Gv^2(t)$

Note limiting cases are of use

If R -> 0 $v(t) = R \cdot i(t) = 0$ for any i(t), a 'short circuit'

If R -> ∞ $i(t) = \frac{v(t)}{R} = 0$ for any v(t) an ‘open circuit’

We now have the elements we need to begin circuit analysis, although we will want to come up with more formal techniques soon.

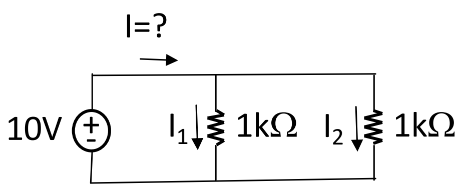


I=V/R

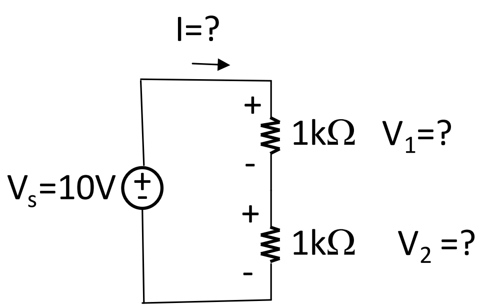
I=10V/1kΩ

I=10mA

Make sure you know and can apply Ohm’s law!



I_1= 10V/1kΩ = 10mA
I_2= 10V/1kΩ = 10mA
Then I =I_1 + I_2 = 20mA (conservation of current)



I= 10V/2kΩ = 5mA
V_1= 5mA X 1kΩ = 5V
V_2= 5mA X 1kΩ = 5V
All current must pass through both resistors.
V_s = V_1 + V_2 conservation of voltage, energy.

P_S = 10V X -5mA = -50mW

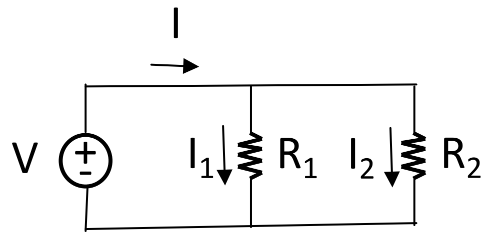
P_1 = $(5mA)^2$ X 1kΩ = +25mW

P_2 = $(5mA)^2$ X 1kΩ = +25mW

Using sign convention $P_s + P_1 + P_2 = 0$

Current Divider

Some circuit configurations are so common we give them special names. One such configuration is called a current divider. In this circuit current is split between two resistors. This happens so often it is worth deriving a couple of equations for this circuit:



We first note that the two currents I1 and I2 can be written in terms of voltage:

$I_1 = V/R_1$ $\quad$ $I_2 = V/R_2$

Next we note that I must be the sum of I_1 and I_2:

$I = I_1 + I_2 = V/R_1 + V/R_2$

Now we ask the question what faction of the total current I flows through R_1? We can write an equation:

$$\frac{I_1}{I} = \frac{\frac{V}{R_1}}{\frac{V}{R_1} + \frac{V}{R_2}} = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{\frac{R_1 R_2}{R_1}}{R_1 + R_2} = \frac{R_2}{R_1 + R_2}$$

Similarly:

$$\frac{I_2}{I} = \frac{R_1}{R_1 + R_2}$$

NOTE: As can be seen there are only a couple of logical steps to derive these two equations. Therefore, they should **<u>NOT</u>** be memorized, but rather re-derived as needed. You will see this again many times over the course of your degree in many courses.

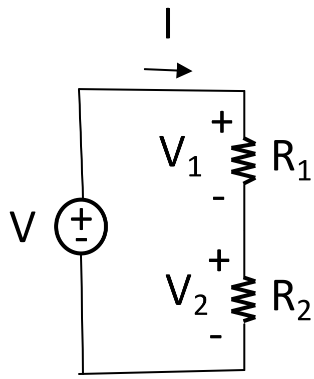
We can also find an equivalent resistance of R_1 and R_2 in this circuit by using Ohm's law again:

$$R_{eq} = \frac{V}{I} = \frac{V}{I_1 + I_2} = \frac{V}{\frac{V}{R_1} + \frac{V}{R_2}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

These two resistors are said to be in parallel. In general any two resistors that have their positive terminals and their negative terminals shorted to each other and therefore always see the same voltage are said to be in parallel.

<u>Voltage Divider</u>

Another very common circuit configuration is called the voltage divider:



We note that the sum of the voltage drops across R_1 and R_2 must equal the voltage supplied by the source V:

$$V = V_1 + V_2$$

We also note that current I must flow through each resistor so $V_1 = I \cdot R_1$ and $V_2 = I \cdot R_2$ therefore:

$$V = IR_1 + IR_2$$

We would like to know now what faction V_1 is of V:

$$\frac{V_1}{V} = \frac{IR_1}{IR_1 + IR_2} = \frac{R_1}{R_1 + R_2}$$

Similarly:

$$\frac{V_2}{V} = \frac{R_2}{R_1 + R_2}$$

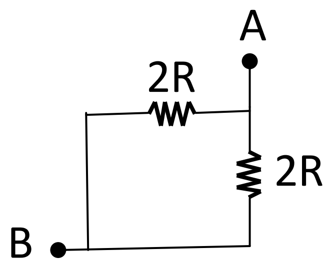
We would also like to know the equivalent resistance of this circuit:

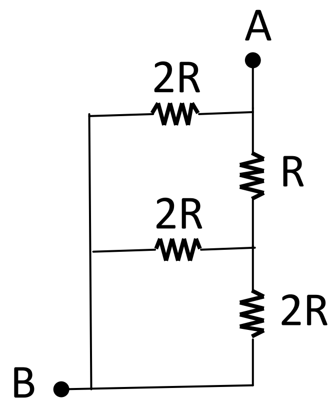
$$R_{eq} = \frac{V}{I} = \frac{V_1 + V_2}{I} = \frac{IR_1 + IR_2}{I} = R_1 + R_2$$

These two resistors are said to be in series. Any two resistors where one terminal of the first resistor is shorted to one terminal of another transistor and that node has no other connections and therefore both resistors MUST draw the same current are said to be in series.

***** Very important skill. You should become very good at quickly identifying when resistors are in series or in parallel.

Practice Exercise: Find the equivalent resistance of each of the following two networks between points A and B:


A
2R
2R
B


A
2R
R
2R
2R
B